Maxwell relation, $\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial s}\right)_P$

2nd

We know, $dH = Tds + VdP$

If P is constant, $dP = 0$

$$dH = Tds$$

$$\text{or } \left(\frac{\partial H}{\partial s}\right)_P = T \quad \text{———} \quad (1)$$

If S is constant, $ds = 0$

from eqn $dH = Tds + VdP$

$$\left(\frac{\partial H}{\partial P}\right)_s = V \quad \text{———} \quad (2)$$

On differentiating eqn (1) w.r.to P where S is constant

$$\frac{\partial^2 H}{(\partial s)_P (\partial P)_s} = \left(\frac{\partial T}{\partial P}\right)_s \quad \text{———} \quad (3)$$

And eqn (2) is differentiating w.r.to S where P is constant

$$\frac{\partial^2 H}{(\partial P)_s (\partial s)_P} = \left(\frac{\partial V}{\partial s}\right)_P \quad \text{———} \quad (4)$$

From equation (3) and equation (4), we have.

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial s}\right)_P \quad \text{———} \quad (5)$$

The above equation (5) is also Maxwell relation.

3rd

Maxwell relation:- $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$

We know $dA = -SdT - PdV$

When T is contant, $dT = 0$

$$\therefore \left(\frac{\partial A}{\partial V}\right)_T = -P \quad \text{——} (1)$$

When V is constant, $dV = 0$

eqn $dA = -Sdt - PdV$

$$\left(\frac{\partial A}{\partial T}\right)_V = -S \quad \text{——} (2)$$

Differentiating equation (1) w.r. to T, where V is constant.

$$\frac{\partial^2 A}{(\partial V)_T (\partial T)_V} = -\left(\frac{\partial P}{\partial T}\right)_V \quad \text{——} (3)$$

Again differentiating equation (2) with respect to V, where T is Const.

$$\frac{\partial^2 A}{(\partial T)_V (\partial V)_T} = -\left(\frac{\partial S}{\partial V}\right)_T \quad \text{——} (4)$$

From equation (3) and (4) we have.

$$-\left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial S}{\partial V}\right)_T$$

or $$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

The above relation is also Maxwell relation.

—x—

Maxwell relation,:- $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

4th

We know $dG = -SdT + VdP$

When T is const, $dT = 0$

$\therefore \partial G = VdP$

$$\left(\frac{\partial G}{\partial P}\right)_T = V \quad \text{——— (1)}$$

$\because dG = -SdT + VdP$

When Pressure P is constant $dP = 0$

$\therefore dG = -SdT$

$$\text{or } \left(\frac{\partial G}{\partial T}\right)_P = -S \quad \text{——— (2)}$$

Differentiating eq (1) w.r.to T where P is const.

$$\frac{\partial^2 G}{(\partial P)_T (\partial T)_P} = \left(\frac{\partial V}{\partial T}\right)_P \quad \text{——— (3)}$$

Differentiating eqn (2) w.r.to P, where T is constant

$$\frac{\partial^2 G}{(\partial T)_P (\partial P)_T} = -\left(\frac{\partial S}{\partial P}\right)_T \quad \text{——— (4)}$$

From equation (3) and equation (4)

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

$$\text{or } \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \quad \text{——— (5)}$$

The above eqn is also Maxwell relation.

5th Maxwell relation, $\left(\frac{\partial U}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_P$

To derive maxwell relation the following two eqn are important.

$$dU = TdS - PdV \quad \text{—— (i)}$$
$$dH = TdS + VdP \quad \text{—— (ii)}$$

Consider eqn $dU = TdS - PdV$

If V is constant, then $dV = 0$

$$dU = TdS$$

$$\text{or} \left(\frac{\partial U}{\partial S}\right)_V = T \quad \text{—— (1)}$$

Consider eqn $dH = TdS + VdP$

When Pressure P is Constant, $dP = 0$

$$dH = TdS$$

$$\text{or} \left(\frac{\partial H}{\partial S}\right)_P = T \quad \text{—— (2)}$$

From equation (1) and equation (2)

$$\left(\frac{\partial U}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_P$$

The above eqn is also Maxwell relation.

6th Maxwell relation :- $\left(\frac{\partial u}{\partial v}\right)_s = \left(\frac{\partial A}{\partial v}\right)_T$

* We know $dU = Tds - Pdv$ — (a)

$dA = -SdT - Pdv$ — (b)

Consider eqn $du = Tds - Pdv$

If S is constant $ds = 0$

$$\left(\frac{\partial u}{\partial v}\right)_s = -P \quad \text{— (1)}$$

From eqn $dA = -SdT - Pdv$

If T is constant, $dT = 0$

$$\therefore \left(\frac{\partial A}{\partial v}\right)_T = -P \quad \text{— (2)}$$

From eqn (1) and (2) $\left(\frac{\partial u}{\partial v}\right)_s = \left(\frac{\partial A}{\partial v}\right)_T$ Proved

* Prove, $\left(\frac{\partial H}{\partial P}\right)_S = \left(\frac{\partial G}{\partial P}\right)_T$

7th We know, $dH = Tds + VdP$

$dG = -SdT + VdP$

When S is constant, $ds = 0$

$\therefore dH = Tds + VdP$

$$\therefore \left(\frac{\partial H}{\partial P}\right)_S = V \quad \text{— (1)}$$

$dG = -SdT + VdP$

When T is constant, $dT = 0$

$$\left(\frac{\partial G}{\partial P}\right)_T = V \quad \text{— (2)}$$

From equation (1) and (2), we have

$$\left(\frac{\partial H}{\partial P}\right)_S = \left(\frac{\partial G}{\partial P}\right)_T$$

It is also Maxwell relation.

8th

Prove. $\left(\frac{\partial A}{\partial T}\right)_V = \left(\frac{\partial G}{\partial T}\right)_P$

Consider eqn $dA = -SdT - PdV$ —— (a)

$dG = -S\,dT + VdP$ —— (b)

Consider eqn $dA = -SdT - PdV$

when V is constant $dV = 0$

$$\left(\frac{\partial A}{\partial T}\right)_V = -S \quad \text{—— (1)}$$

Consider eqn $dG = -S\,dT + VdP$

when P is Constant, $dP = 0$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S \quad \text{—— (2)}$$

from eqn (1) and (2)

$$\left(\frac{\partial A}{\partial T}\right)_V = \left(\frac{\partial G}{\partial T}\right)_P$$

Proved.